



Model to predict overheating risk based on an electrical capacitor analogy

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Abstract

It is important for building designers to be able to judge if a space is likely to overheat, whether to determine if HVAC plant is required or to design this out by passive means. To this end several overheating risk criteria have recently been published. These tend to be based on some limiting number of occasions that an indoor temperature may be exceeded. Somewhat disconcertingly, these criteria are not based on a systematic analysis of the causes of overheating, such that the thresholds used are essentially arbitrary.

Based on analogy between the charging and discharging of humans' tolerance to overheating stimuli and that of charge in an electrical capacitor, this paper proposes a simple mathematical model for predicting overheating risk given a set of measured/simulated environmental conditions. The model is analytically based, but uses coefficients (α , β) to empirically tune its charging/discharging time constants to a given population and situation. Comparisons with results from a dedicated field survey conceived to help develop and test the model are very encouraging, but scope for further improvement is discussed.

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1. Introduction

With the desire to minimise energy consumption and associated harmful atmospheric emissions, designers of European buildings increasingly strive to avoid the use of mechanical cooling by passive means. This typically involves minimising excess casual and solar heat gains as well as heat conduction from outside and discharging the heat that has accumulated within exposed fabric during the daytime by nocturnal ventilation (night cooling). During this process, it is important that designers have some basis for judging whether or not the need for air conditioning has been successfully eliminated, so that the (passive) design can be further improved or so that the necessary mechanical system can be designed.

Such decisions have conventionally been made on the basis of comparing predictions of instantaneous thermal satisfaction, for example based on the steady state model of Fanger [1], with a static thermal comfort envelope. Such envelopes have evolved

from the narrowly defined ASHRAE 55-1966 comfort envelope, which spans a temperature interval of just ~ 2 °C and a relative humidity interval of 20–80%, to one which is seasonally dependant, broader and accounts for limiting wet bulb temperatures: ASHRAE 55a-1996; see Fiala et al. [2] for further discussion.

The seasonal dependence of ASHRAE 55a:1996 reflects the growing acceptance of adaptive algorithms, such as those of Humphreys [3], Humphreys and Nicol [4], Nicol and Humphreys [5], de Dear and Brager [6], etc. Indeed, this has prompted a new generation of standards that define comfort envelopes for free-running buildings based on indoor temperatures which vary as a function of different time periods of outdoor temperature. Examples include the Dutch standard ISSO 74:2005 [7], the US standard ASHRAE 55:2004 [8] and the proposed European standard prEN 15251:2005 [9]. Only the proposed International standard prEN ISO 7730:2005 [10] continues to ignore adaptation of occupants' comfort temperature as a function of outside conditions (except in that clothing level is assumed to be seasonally dependent). Pfafferoth [17] provides a synopsis of these as well as standards that relate more to buildings with HVAC.

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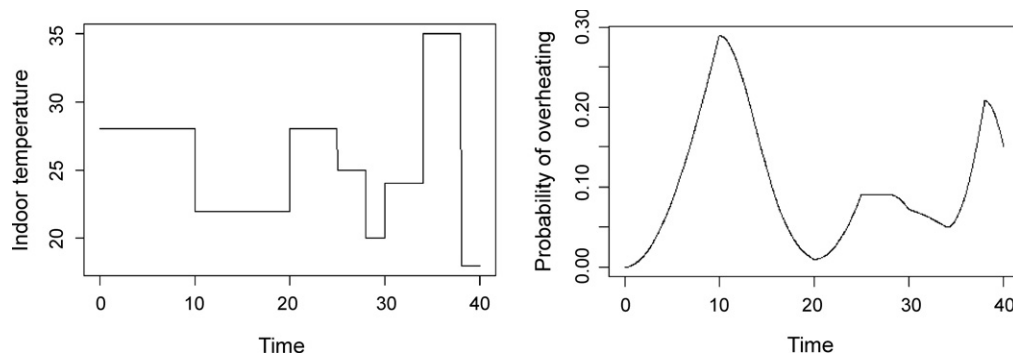


Fig. 1. Step changes in indoor air temperature (left) and a hypothetical overheating probability response curve (right).

The Dutch standard ISSO 74:2005 goes rather further than the others by defining two different types of envelope for buildings which present occupants with either little or relatively good adaptive control possibilities. This is in line with Baker and Stendeven’s [11] suggestion that occupants’ range of thermal neutrality increases with the adaptive opportunities available to them. More or less relaxed boundaries are also proposed for each envelope type, their extent depending upon occupants’ comfort expectations for a particular building type/activity (e.g. existing building [C], standard office [B] or office headquarters [A]). But as with the alternative standard above, the appropriate envelope must be strictly adhered to (see Ref. [12]).

This is a somewhat surprising departure from the previous work of Van der Linden et al. [13] which defined a limiting number of times during which a comfort envelope (defined by a predicted mean vote (PMV) of ± 0.5 , based on the Fanger [1] model) may be exceeded.¹ This follows from Baker and Stendeven’s [11] suggestion that occupants are tolerant of occasional departures from thermal neutrality—up to some limit. Other guidelines inspired by this work include that of the Canton of Zürich in Switzerland which allows for 30 degree-hours above a limiting temperature curve that varies with outside temperature and a UK [14] guideline which suggests that resultant temperatures may exceed 25 °C or 28 °C for 5% or 1% of occupied hours, respectively.

However, these latter standards/guidelines are based on a-priori hypotheses which have not been rigorously tested. No rigorous empirical study has been conducted to confirm that overheating results from an accumulation of heat stress events or to define what the corresponding limits are. This paper reports on a study which was conceived to resolve this problem. In the next section we discuss the hypotheses on which the model is based and proceed to describe the formulation of the model. We then describe the field survey campaign that was employed to support its development and we present the associated results with which the model is compared. We finally

discuss ways in which the model might be further improved in the future.

2. Model formulation

Following from the above rationale that it is an accumulation of heat stimuli that leads us to consider a space as having overheated, the principal hypothesis of our model is that (the storage of) human tolerance to overheating stimuli can be considered as equivalent to the storage of electrical charge in a capacitor. By this it is meant that during a series of particularly warm days this overheating tolerance is discharged. If this is followed by a cooler period then this tolerance is recharged. An extreme example of this is a recharging of our tolerance during winter-time in readiness for discharging during the following summer. If tolerance $T \in [0,1]$ then the probability of overheating $P_{OH} = 1 - T$.

Now, let us consider the well-known situation of a circuit including a capacitance C and a resistance R , receiving a constant voltage U_0 . The voltage in these components are linked by the equation $U_0 = U_R + U_C$, which leads to the differential equation: $RC(dU_C/dt)(t) + U_C(t) = U_0$. Setting $U_{C0} = U_C(t = 0)$, we obtain the well-known solution for the discharging of the capacitor: $U_C(t) = U_{C0} \exp(-t/RC)$ and for its corresponding charging we have: $U_C(t) = U_0(1 - \exp(-t/RC)) + U_{C0} \exp(-t/RC)$; where the product RC is a (dis)charging time constant τ . In the context of our overheating model, we shall set $U_0 = 1$ and interpret $U_C(t)$ as the probability of having overheated $P_{OH}(t)$ due to an accumulation of heat stimuli and $U_R(t)$ as the tolerance to these stimuli, so that $U_R(t) + U_C(t) = 1$ for any time t . Application of this reasoning is illustrated in Fig. 1.

We assume that the heat stimuli may be represented by the excess of some reference temperature θ^* , say 25 °C. We therefore replace the reciprocal of our time constant $1/\tau$ with the time-varying expression $\alpha(\theta(t) - \theta^*)$ in the charge expression, given a constant α , and with $\beta(\theta^* - \theta(t))$ in the discharge expression.

Considering now the time interval $[t - \Delta t, t]$ we have the following expression for the probability of overheating as a function of $P_{OH}(t - \Delta t)$ which, whilst charging, is

$$P_{OH}(t) = 1 - \exp(-\alpha(\theta(t) - \theta^*)t)(1 - P_{OH}(t - \Delta t)) \quad (1a)$$

¹ With hours even being weighted according to the magnitude of excess (e.g. 1 h with a predicted percentage of dissatisfied (PPD) occupants of 20% counts for twice as much as 1 h with a 10% PPD).

and during discharging is

$$P_{OH}(t) = \exp(-\beta(\theta^* - \theta(t))t)(P_{OH}(t - \Delta t)) \quad (1b)$$

with α , β being parameters to determine empirically.

Let us now consider n consecutive periods of charge $[t_0, t_0 + \Delta t], \dots, [t_n - \Delta t, t_n]$ during the period $[t_0, t_n]$. Using Eq. (1a), inserting a similar expression for $P_{OH}(t - \Delta t)$ and solving recurrently until t_0 , we have for charging, that:

$$P_{OH}(t_n) = 1 - \exp\left(-\alpha \sum_{i=0}^{n-1} [(\theta(t_i - i\Delta t) - \theta^*)(t_i - i\Delta t)]\right) \times (1 - P_{OH}(t_0)) \quad (2)$$

Now, if we define degree-hours (DH) of heat stimuli above our reference temperature during the period $[t_i, t_j]$ as $DH = (\theta(t_j) - \theta^*)(t_j - t_i)$ and furthermore reason that $P_{OH}(t_0) = 0$ then we obtain the following for charging:

$$P_{OH}(t_n) = 1 - \exp\left(-\alpha \sum_{i=1}^n DH_{t_0, t_i}\right) (1 - P_{OH}(t_0)) \quad (3a)$$

And for discharging during n consecutive periods $[t_0, t_0 + \Delta t], \dots, [t_n - \Delta t, t_n]$, we have:

$$P_{OH}(t_n) = \exp\left(-\beta \sum_{i=1}^n DH_{t_0, t_i}^*\right) P_{OH}(t_0) \quad (3b)$$

where in this case $P_{OH}(t_0)$ refers to the probability of overheating at the time of transition from charging to discharging of our human capacitor and $DH_{t_0, t_i}^* = (\theta^* - \theta(t_0))(t_i - t_0)$. Thus we are able to model a temporal evolution of the probability of overheating within the limits $[0,1]$ during an arbitrary time period.

3. Comparisons

To support development and testing of the overheating model described above, a thermal comfort/overheating field survey campaign was conducted during the summer of 2006. In the following section we present briefly the methodology upon which this campaign was based. This is followed by a synopsis of some of the results from this campaign. Then we present a simplification of the above model, the derivation of the associated empirical coefficients and compare measurements with corresponding predictions.

3.1. Field survey methodology

A total of eight non-air-conditioned office buildings, each located in Switzerland, were selected based on a desire to have a reasonable diversity in terms of their design concept and the adaptive opportunities available to occupants. For reasons of practicality, these were all located within a 50 km radius of Lausanne (latitude 46.5°N, longitude 6.7°E). For each building, a three-stage field survey methodology was employed:

- **Initial transverse questionnaire:** administered to a large proportion of building occupants during the same day to gauge general satisfaction with the thermal environment and the availability and effectiveness of means to control the environment. A total of 257 responses were obtained from the eight buildings under investigation.
- **Longitudinal questionnaire:** electronic questionnaire administered at regular intervals on a daily basis to a sub-set of volunteer occupants of each building, to produce time-series data regarding participants' thermal satisfaction, personal characteristics (clothing and activity level) and interactions with their environment; with simultaneous recording of environmental conditions (Fig. 2a). Most crucially participants were asked to press an 'overheated' button once they

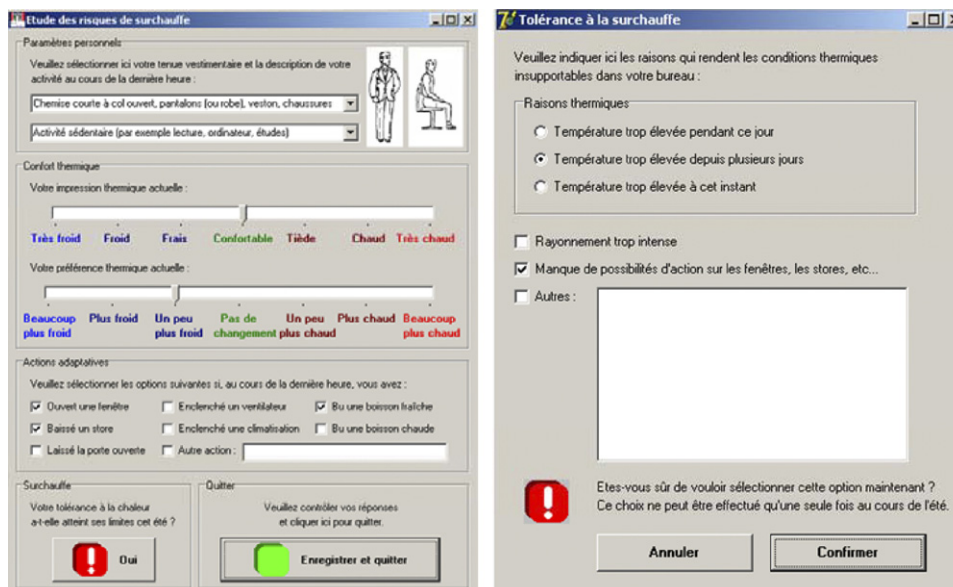


Fig. 2. Longitudinal e-questionnaire (a: left) and dialogue box for explanation of cause of overheating (b: right); in French.

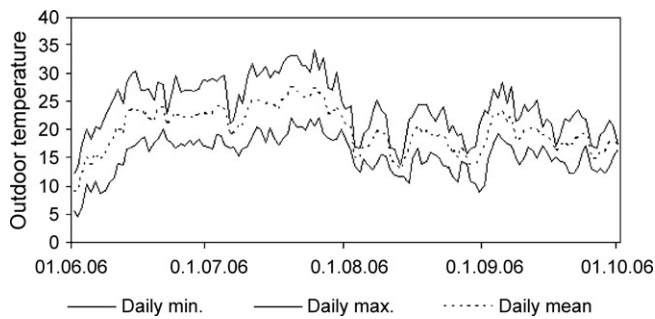


Fig. 3. Temporal profile of average daily minimum/mean maximum temperature for the three sites.

felt that their thermal tolerance had been surpassed. To ensure that we would obtain unambiguous information participants were asked to select this option one time only. After confirming their choice, this prompted a further dialogue box (Fig. 2b) which requested participants to describe the circumstances that led to this transition in tolerance (i.e. from the space having been thermally acceptable to now having overheated). More specifically whether this was caused by an excessive temperature during the moment in question or whether the temperature was excessively high throughout the day in question or throughout several prior days. Participants were also asked to remark whether this overheating was influenced by excess solar gain, a lack of control options or to offer alternative explanations. In total there were 60 participants in this study, who produced a total of 5908 responses (i.e. each participant completed the questionnaire an average 98 times).

- **Final transverse questionnaire:** At the end of the longitudinal study a short second transverse questionnaire was administered to ascertain whether the population at large perceived their building to have overheated and if so to explain why. A total of 206 responses were obtained.

3.2. Field survey results summary

The longitudinal study lasted from June 13th until September 27th 2006. From mid June until the end of July 2006, most of Europe experienced a heat wave (Fig. 3). It was during this period that all occurrences took place of the button being pressed indicating that participants' tolerance had been exceeded (their environment has now become overheated).

Of the 60 participants only 22 pressed this button.² Of these only 18% indicated that this was caused by excess temperature at the moment in question, whereas 27% and 55% indicated that this was due to excess temperature during the day in question and during the past several days, respectively.

This result tends to confirm the hypothesis that occupants are tolerant of occasional departures from comfort; or more specifically that overheating is due to an accumulation of heat stress events rather than a single event (which we may regard as

a special case of an accumulation of tolerance discharge over a very short period of time (e.g. large α).

Recipients of this questionnaire were also asked whether they were generally satisfied with their thermal environment and also whether they felt that their office environment had overheated during the summer of 2006. For the eight buildings studied, on average 45% of occupants were thermally satisfied with their environment, but some 74% perceived their buildings to have overheated—considerably higher than those that participated in the longitudinal study, perhaps suggesting that occupants are less tolerant on reflection than at the time of the event.

3.3. Model application and verification

During the warmest period of our field survey campaign in which all overheating events were logged, no internal air temperature below 25 °C was recorded during working hours. The period was thus a continuously charging period of overheating probability (or inversely a discharging of tolerance). Under such (continuously charging) circumstances we are able to simplify Eq. (3a) somewhat, so that

$$P_{OH}(t) = 1 - \exp(-\alpha' DH_{0,t}) \tag{4}$$

where $\alpha' = \alpha \Delta t$. Determining α' is then performed by the fitting of data using the regression equation $\log(1 - P_{OH}(t)) = -\alpha' DH_{0,t}$, where P_{OH} is the observed cumulative probability of overheating. The corresponding regression on our data (Fig. 4) gives a value of $\alpha' = 4.75 \times 10^{-4} [K^{-1} h^{-1}]$ with good agreement ($r^2 = 0.98$), albeit perhaps based on a limited range of $0 < (DH > 25^\circ C) < 1200$.

The corresponding predicted probability of overheating using this simplified form of the model (i.e. for continuous charging) is compared with the measurement results in Fig. 5.

Note that this new model predicts the probability that a population will perceive a given space to have overheated, but we may also interpret this as the proportion of a given population that will perceive a space to have overheated.

Now, to test our model thoroughly we would ideally have data spanning the entire range $P_{OH} \in [0,1]$, but unfortunately

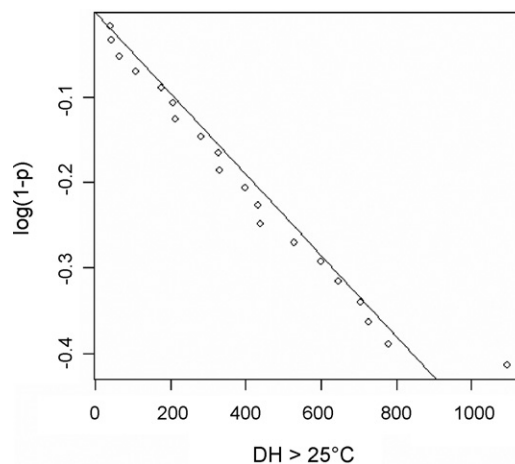


Fig. 4. Linear regression for derivation of α' .

² And amongst these, five did so almost immediately, suggesting that this may have been a mistake.

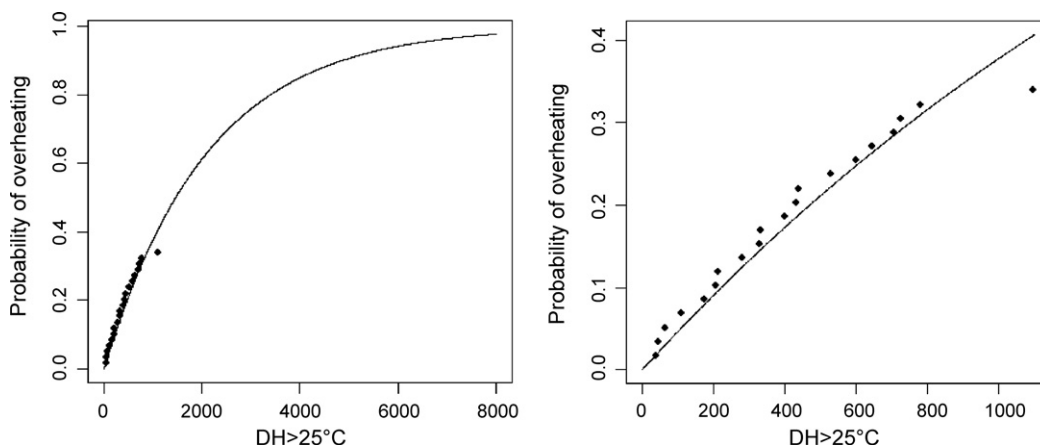


Fig. 5. Predicted (solid line) and measured (solid diamonds) overheating probability as a function of $DH > 25^\circ\text{C}$ in the probability range $[0,1]$ (left) and focussing on the range $[0,0.4]$ (right).

the summer of 2006 was not sufficiently warm to provide us with these data. Nevertheless we know that building designers seek to satisfy the comfort expectations of the majority of a building population, so that current thermal comfort standards target a PPD of $\leq 20\%$. We may therefore suggest that our model is valid within the range of practical application [i.e. it compares well with measurements in the range $0 \leq P_{OH} \leq 0.2$ that is of interest to building designers].

For an example of the temporal evolution of overheating probability using this model given a corresponding thermal stimulus, the reader is referred back to Fig. 1 above,³ for which a reference temperature of 25°C was used to calculate the degree-hours of overheating stimuli.

4. Discussion

In expressing overheating probability as the inverse of tolerance to overheating stimuli which discharges with temperatures above a certain limit and (re)charges below another limit, based on analogy with an electrical capacitor, we have a coherent basis for integrating overheating stimuli (rather than the current practice of simply counting the number of occasions that a certain comfort limit is surpassed) and the way in which human tolerance to such influences is diminished and replenished during and between periods of warm weather. But this model could be criticised for not considering other physical (e.g. radiant temperature, air speed and relative humidity) or personal (clothing and activity level) parameters or indeed the

impact of occupants’ ability to control their environment on their tolerance to overheating stimuli and thereby on the probability of overheating.

But in a well-designed indoor environment the radiant temperature should be relatively close to air temperature and the air speed should be restricted to within reasonable bounds.⁴ Furthermore, in summer time it is reasonable to assume that occupants are similarly lightly clothed (a certain minimum clothing level must be maintained) and that they restrict their metabolic activity to a sensible minimum (office occupants tend in any case to have a somewhat invariant sedentary activity level).

The issue of adaptation is perhaps most pertinent. As Baker and Stenden eloquently suggested in 1996, “if a change occurs such as to produce discomfort, people react in ways which tend to restore their comfort”. Furthermore, when available adaptive opportunities are exercised, these tend both to reduce discomforting stimuli (e.g. shading to reduce radiation absorption; window openings to increase convective cooling, etc.) and to affect “cognitive tolerance” to these stimuli: so that when the cause of discomfort is understood (and indeed partially the occupant’s responsibility) then tolerance is increased. These physiological and psychological adaptive processes are suggested to increase occupants’ summertime neutral temperature (the temperature at which they feel neither warm nor cool), with consequent implications for overheating risk.

This notion of adaptability is implicit in our model, in the sense that it has been tuned to a set of buildings within which occupants have some degree of control over their environment. To take this into account in an explicit way, and thereby produce a more general form of model, we have two options. One option would be to define different sets of coefficients α , β for buildings with different degrees of adaptability. An alternative (and more useful approach) would be to provide a means for adjusting α , β so that the effects of adaptive processes (elevation of neutral temperature due to adaptive processes) in our sample of buildings are removed [so that $DH_{t_0,t}$ in the regression $\alpha' = -\log(1 - P_{OH}(t))/DH_{t_0,t}$ is adjusted] together with a means of calculating degree-hours of overheating for use

³ Note that to produce this diagram we use our empirically derived value for $\alpha' = 4.75 \times 10^{-4}$ and an estimate for β' . For this latter we hypothesise that $\beta \gg \alpha$ because we suppose that occupants are psychologically more sensitive to the thermal relief to overheating stimuli that they feel during a cool period which follows a warm period in which their tolerance is gradually discharged. In the absence of measured results we assume, purely for illustrative purposes, that $\beta' \approx 10\alpha'$.

⁴ The absence of consideration of humidity however may lead to a real geographical limitation of the model.

in the equation $P_{OH}(t) = 1 - \exp(\alpha' DH_{t_0,t})$ which then accounts for building-specific adaptive processes in an explicit way. One approach to achieve this would be to adjust the temperature used to calculate $DH_{t_0,t}$ by using the empirical adaptive increments of Haldi and Robinson [15].

On a related note, our field survey design raises some interesting questions, such as: does the process of observation influence participants' perceptions? Is it sufficient to ask participants to vote for overheating just once during the study?

Concerning the latter point, participants may be inclined to delay their vote until they are unequivocally sure that their tolerance has been surpassed (since they are unable to predict which conditions are likely to follow). This was considered to be an acceptable risk, since our key concern was to have a single clear and decisive vote that "yes, I perceive this space to have now overheated", accompanied by the thermal history that led to this vote, rather than to have multiple votes and associated histories and not to know upon what to base our analysis.

The former issue is also a potential flaw in our experimental design—it is possible that the more we ask a person to reflect upon a particular environmental situation, the more sensitive that person becomes to any associated discomforting influences. However, this effect may tend to cancel the other noted above.

5. Conclusions

In this paper we have attempted to define and verify a new model to predict overheating risk given a set of measured/simulated environmental conditions. This model is based on analogy between the (dis)charging of human tolerance to overheating stimuli and of charge in an electrical capacitor. In this the model is analytically based, but uses coefficients α , β to empirically tune the model's charging/discharging time constants to a given population and situation. The results from a first application of this model are very encouraging. However, there is scope for further improvement:

- The current form of the model considers only a single parameter: indoor air temperature. A more elaborate model might consider some more holistic physical measure such as (degree-hours of) operative temperature.
- The model has been developed for a temperate climate in which the range of temperature and humidity is moderate, so that the model's applicability to warm–humid climates may be questionable. It might be that empirical coefficients should depend on the climate type.
- The model does not presently consider, in an explicit way, the influence of adaptive processes on (dis)charging time constants, but a mechanism for resolving this has been identified.

Furthermore, all of our overheating events took place during a period of constant discharging of tolerance to elevated temperatures, so that it has not yet been possible for us to

calibrate our empirical coefficient β . For this we would need data which contains at least one instance of a cool period between two periods which were warm enough to cause people to vote that an overheating event had taken place.

The authors would be pleased to collaborate with other research groups to enlarge the field survey database to facilitate model improvements addressing the above to be tested.

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